Two-phase flow: flow patterns / models

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Summary

• Flow patterns
  ✓ Flow pattern map
  ✓ Prediction of flow pattern

• Models of two-phase flow
  ✓ Conservation equations
  ✓ Closure problem
  ✓ Various models
  ✓ Drift flux model
  ✓ Two-fluid models
Example of flow pattern in a 5 cm diameter pipe with air and water

\[ j_k = \frac{Q_k}{A} \]

is the flux (or superficial velocity) of phase \( k \)

where \( Q_k \) is the volumetric flow rate and \( A \) the pipe cross-section

**Prediction of flow pattern**

- **Position of the problem:**
  - For given fluid properties (viscosities, densities and surface tension),
  - given geometries (pipe diameter and slope),
  - given flow rate or superficial velocity of each phase (\( Q_L, Q_G \))
  - determine the flow pattern that would occur.

- **Two different methods exist for predicting the flow pattern**
  - The flow pattern is imposed from a physical criterion of existence
  - The flow pattern is determined from the minimisation of the gas fraction

- **There is no reason that the two methods lead to the same results**
Prediction of flow pattern: physical criterion of existence

• Example of the transition between stratified and slug flow:
  ✓ Stratified flow maintains unless it is destabilized by Kelvin-Helmholtz instability.
  ✓ The stability may be determined:
    ➢ Either from a linear stability analysis of stratified flow (discussed in another lesson)
    ➢ Or by a semi-empirical correlation, e.g. the Taitel & Dukler stability criterion.
    According to this criterion, waves at the interface will grow to form slug flow if:

\[
U_g - U_l > \sqrt{\frac{g \cos \theta \Delta \rho h_G}{\rho_G}}
\]

The method requires that the physical quantities \((U_l, U_g, h_G)\) be determined in the stratified flow pattern to check whether the criterion is fulfilled or not.

Prediction of flow pattern: minimisation of the gas fraction \(R_G\)

• Example of the transition between stratified and slug flow:
  ✓ To ensure the continuity of the gas (or liquid) fraction across the transition one may assume that the flow pattern that occurs is the one that minimize (or maximize) the gas fraction (or any other kinematic quantity; e.g. phase velocity)
  ✓ The flow pattern may be determined by:
    ➢ Calculating \(R_G\) for stratified flow and for slug flow
    ➢ Choosing the flow pattern that leads to the smaller value of \(R_G\).
Prediction of flow pattern: summary

- Example of the transition between stratified and slug flow:
  - Method using a physical criterion of existence:
    1. Calculation of the properties of stratified flow
    2. Verification of the criterion of existence for stratified flow
  - Method from the minimisation of the gas fraction $R_G$:
    1. Calculation of the properties of stratified flow
    2. Calculation of the properties of slug flow

Summary

- **Flow patterns**
  - Flow pattern map
  - Prediction of flow pattern
- **Models of two-phase flow**
  - Conservation equations
  - Closure problem
  - Various models
  - Drift flux model
  - Two-fluid models
Models of two-phase flow: conservation equations

- **Mass conservation**
  \[
  \frac{\partial}{\partial t} (\rho_k R_k) + \frac{\partial}{\partial z} (\rho_k R_k U_k) = 0
  \]
  for steady full developed flow
  \[R_k U_k = j_k\]

- **Momentum conservation**
  \[
  \frac{\partial}{\partial t} (\rho_k R_k U_k) + \frac{\partial}{\partial z} (\rho_k R_k U_k U_k) =
  \]
  \[
  -\frac{\partial}{\partial z} (R_k P_k) + F_{wk} + F_{lk} - \rho_k R_k g \sin \theta
  \]
  for steady full developed flow
  \[0 = -R_k \frac{dP}{dz} + F_{wk} + F_{lk} - \rho_k R_k g \sin \theta\]

Models of two-phase flow: closure problem

- **Set of equations:**
  \[R_k U_k = j_k\]
  \[0 = -R_k \frac{dP}{dz} + F_{wk} + F_{lk} - \rho_k R_k g \sin \theta\]

- **Set of governing equations:**
  \[R_k U_k = j_k\]
  **mass conservation**
  \[0 = \frac{F_{wk}}{R_k L_k} + \frac{F_{lk}}{R_k L_k} + \frac{F_{ul}}{R_k L_k} + (\rho_k - \rho_l) g \sin \theta\]
  **Equation of phase fraction**
  \[R_k + R_l = 1\]
  **Geometrical condition**
  \[F_{lG} + F_{LG} = 0\]
  **Dynamical condition**
Models of two-phase flow: various models

- The **two-fluid model**, so called because it solves the momentum equations for each phase. The resulting phase fraction equation is an implicit equation that requires an iterative method:

  \[
  0 = \frac{F_w(R, U, G)}{R} - \frac{F_f(R, U, L)}{R} + \frac{F_{fg}(R, U, G)}{R} + \Delta \rho g \sin \theta
  \]

- The **drift flux model**, so called because the above equation is replaced by a physical correlation that gives \( R - U \) versus the flow conditions (flux, physical properties, pipe diameter and slope).

- **Rule:**
  - Separated flow requires the two-fluid model
  - Bubbly flow may be solved either by using one or the other
  - Slug flow is solved with a specific model (discussed in another lesson)

Models of two-phase flow: drift flux model (vertical bubbly flow)

- For fully developed steady flow

  \[
  R_L U_L = j_L \\
  R_G U_G = j_G
  \]

- We need a closure law of the form:

  \[
  f(R_G, U_L, U_G) = 0
  \]

- Definition of the drift flux

  \[
  j_{LG} = R_G (U_G - j)
  \]

  where: \( j = j_L + j_G \)

- Property of the drift flux

  \[
  j_{LS} = R_L R_G (U_G - U_L)
  \]
Models of two-phase flow: drift flux model (vertical bubbly flow)

- **Properties of the law:**
  
  \[ f(R_G, U_L, U_G) = 0 \]
  
  ✓ If we replace \( U_L \) (or \( U_G \)) by \( j_{LG} \) then the law must be invariant in a Galilean transformation. Thus
  
  \[ j_{LG} = F(R_G) \]
  
  ✓ The drift flux must vanish for \( R_G = 0 \) and thus may be expressed under the form of a Taylor series in \( R_G \)
  
  \[ j_{LG} = V_0 R_G + V_1 R_G^2 + \ldots \]
  
  ✓ \( V_0 \) must satisfy the condition:
  
  \[ V_0 = \lim_{R_G \to 0} \frac{V_B - U_L}{R_G} = \lim_{R_G \to 0} \frac{U_G - j}{R_G} \]
  
  ✓ This limit is known: this is rise velocity of a bubble in still liquid
  
  \[ V_0 = V_B \]

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Models of two-phase flow: drift flux model (vertical bubbly flow)

- **Averaged liquid velocity**
  
  \[ U_L = (1 - \varepsilon)U_{LB} + \varepsilon U_G \]

- **Fraction of entrained liquid**
  
  \[ \varepsilon = C_e R_G \]

- **Velocity in still liquid**
  
  \[ V_B = U_G - U_{LB} \]

- **Drift flux**
  
  \[ j_{LG} = V_B R_G (1 - R_G)(1 - C_e R_G) \]

✓ The coefficient \( C_e = 0.5 \) for a bubble in viscous flow

Picture of bubbly flow: part of the liquid is entrained by bubbles

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Rise velocity $V_2$ of a bubble in still liquid

$V^* = F\left[d^*, Mo\right]$,

with

$V^* = \frac{V_B}{\sqrt{g \frac{\rho_f}{\rho_l}}} = \frac{\gamma We^2}{Eo}$

and $Mo = \frac{g \rho_f \gamma v^*}{\sigma} = \frac{Eo We^2}{Re^4}$

Two-phase flow: drift flux model (vertical bubbly flow)

### Models of two-phase flow:

- **Drift flux model (vertical bubbly flow)**

  - **Spherical cap bubbles**
  - **Ellipsoidal bubbles**

\[
\begin{align*}
    d^+ &= d/\sqrt{\frac{\sigma}{\rho_f g}} \\
    V^* &= V_B^2 \sqrt{\frac{\rho_f}{\rho_l}}
\end{align*}
\]

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### Rise velocity $V_B$ of a bubble in still liquid

- **Stokes limit**

\[
\begin{align*}
    V_B &= \frac{4}{3} \frac{\gamma We^2}{Eo} \\
    Mo &= 10^{16}
\end{align*}
\]

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Models of two-phase flow:

- Drift flux model (vertical bubbly flow)

\[ J_{LG} = R_G (U_G - j) = j_G - R_L (j_G + j_L) \]

Co-current up-flow

\[ -j_L / V_B = j_G / V_B - R_G (1 - j_G) (1 - C_e R_G) \]

Counter-current flow

Models of two-phase flow:

two-fluid model (stratified flow)

- How the phase fraction equation look like
  - \( F_w, F_k \) are forces by unit volume exerted by the wall (resp. the other phase)
  - As the wetted perimeters are function of the liquid fraction, the shear stresses at the wall and at the interface remain to be determined
  - Example of simple stratified flow between two plate will be treated

\[ 0 = \frac{F_w G}{R_G} - \frac{F_L L}{R_L} + \frac{F_L G}{R_G} - \frac{F_L L}{R_L} + \Delta \rho g \sin \theta \]

\[ 0 = \frac{S_{wG} \tau_{wG}}{A R_G} - \frac{S_{L G} \tau_{L G}}{A R_G} + \frac{S_{G L} \tau_{G L}}{A R_G} + \Delta \rho g \sin \theta \]
Models of two-phase flow: two-fluid model (stratified 2D flow)

- **Remarks**
  - Two-phase stratified flow is the addition of two single-phase flow problems rather than a two-phase flow problem.
  - Its solution is analytical or at least exact for a flat interface (smooth) and laminar flow regime.
- **Let us consider the Couette-Poiseuille flow problem**
  - Let \( \tau_g \) and \( \tau_h \) be the shear stresses exerted by each plate over the fluid
  - How they relate to the mean velocity

\[
U = \frac{1}{h} \int_0^h u \, dy
\]

- **Couette-Poiseuille flow: momentum equations for fully-developed steady flow**
  - The driving force is composed of the pressure gradient and the gravity contribution
  - The shear stress within the flow is linear with respect to the transverse coordinate

\[
0 = \frac{\partial p}{\partial z} - \rho g \sin \theta + \frac{d\tau}{dy}
\]

\[
0 = \frac{\partial p}{\partial y} - \rho g \cos \theta
\]

\[
a = \frac{\partial p}{\partial z} - \rho g \sin \theta \quad \Rightarrow \quad \tau = -ay - \tau_0
\]

0>0 for downward flow
Models of two-phase flow: two-fluid model (stratified 2D flow)

- If the flow is laminar:
  \[ \tau = \mu \frac{du}{dy} \]
  \[ u = \frac{a y^2}{2\mu} + \left( \frac{ah}{2\mu} + \frac{U_h - U_0}{h} \right)y + U_0 \]

  ✓ 2. Couette flow
  ✓ 3. free surface flow
  ✓ 4. Poisueille flow

Models of two-phase flow: two-fluid model (stratified 2D flow)

- How does the shear stresses relate to velocities:
  ✓ Mean velocity
    \[ U = \frac{ah^2}{12\mu} + \frac{1}{2}(U_0 + U_h) \]
  ✓ Shear stress exerted by the lower wall on the fluid
    \[ \tau_0 = -\frac{h}{h} (6U - 4U_0 - 2U_h) \]
  ✓ Shear stress exerted by the upper wall on the fluid
    \[ \tau_h = -\frac{h}{h} (6U - 2U_0 - 4U_h) \]
  ✓ Specific cases
    \[ \tau_0 = -\frac{h}{h} U \] (Poiseuille flow),
    \[ \tau_0 = -\frac{h}{h} 2U \] (free surface flow).
Models of two-phase flow:
two-fluid model (stratified 2D flow)

• Conservation equations
  ✓ Mass
  ✓ Momentum
  From which one obtains the hold up equation

\[ Q_k = U_i h_i \]
\[ 0 = - h_i \left( \frac{dP}{dh} + \rho_i g \sin \theta \right) + \tau_{ik} + \tau_{ik} \]
with \( \sum_{k=L,G} \tau_k = 0 \)

\[ \Delta \rho g \sin \theta = \frac{\tau_{ik} - \tau_{ik}}{h_L} - \frac{\tau_{ik} + \tau_{ik}}{h_G} \]

Models of two-phase flow:
two-fluid model (stratified 2D flow)

• Closure problem
  ✓ Phase fraction equation

\[ \Delta \rho g \sin \theta = \frac{\tau_{ik} - \tau_{ik}}{h_L} - \frac{\tau_{ik} + \tau_{ik}}{h_G} \]

• From the single-layer problem we have

\[ \tau_{ik} = - \frac{\mu_k}{h_i} (6U_i - 2U_j), \quad \tau_{ik} = - \frac{\mu_k}{h_i} (6U_j - 4U_i) \]

\[ U_i = 3 \left( \frac{\mu_k}{h_i} U_L + \frac{\mu_k}{h_G} U_G \right) \left( \frac{\mu_k}{h_i} + \frac{\mu_k}{h_G} \right) \]

• Equating the stresses at the interface (liquid / gas)

\[ \tau_{ik} = - \frac{3 \mu_k}{h_i} U_i \]
\[ \tau_{ik} = - \frac{\mu_k}{h_i} \left( \frac{U_G - U_L}{h_G + h_i} \right) \]

• Putting into the above expressions of stress leads to

\[ \Delta \rho g \sin \theta = \frac{\tau_{ik} - \tau_{ik}}{h_L} - \frac{\tau_{ik} + \tau_{ik}}{h_G} \]

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Models of two-phase flow: two-fluid model (stratified 2D flow)

- Remark about the specific property of gas-liquid flow: the gas viscosity is generally much smaller than the liquid, thus the symmetry of the closure laws is broken
  - The interfacial velocity is controlled by the liquid motion
    \[ U_i = \frac{3}{2} \frac{\mu_G U_i + \mu_L U_o}{R_L + \frac{\mu_G}{R_G}} \]
  - The shear stress in the gas is almost the same than in a duct of same cross-sectional area
    \[ \tau_{wG} \approx -\frac{3}{2} \mu_G h_G U_G \]
  - The shear stress in the liquid is almost the same than in a free surface flow
    \[ \frac{\tau}{\mu_L} \approx \frac{1}{3} \frac{\mu_L h_L U_L}{R_L} \]

- Solution of the two layers problem with imposed flow rates:
  - The phase fraction equation is an implicit equation in \( R_k \)
  - Introduction of dimensionless numbers
    - Three numbers are needed (they appear in the hold-up equation)
    - Hold-up equation in dimensionless form
    - For negligible gas viscosity, the solution of the dimensionless hold-up equation reduces to \( F(R_i, X, Y) = 0 \)
  - Martinelli parameter
    \[ X = \frac{\mu_G h_G}{\mu_L h_L} \]
  - Gravity to viscous force ratio
    \[ Y = \frac{(\rho_L - \rho_G) g h^2 \sin \theta}{\mu_L h_L} \]
  - Viscosity ratio
    \[ N_\mu = \frac{\mu_G}{\mu_L} \]

- If \( \mu_G \ll \mu_L \):
  \[ U_i \approx \frac{3}{2} U_L \]

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Models of two-phase flow: two-fluid model (stratified 2D flow)

\[ X = \frac{\mu_G}{\mu_L} \]
\[ Y = \frac{(\rho_L - \rho_G)gh \sin \theta}{\mu_L J} \]

Gravity to viscous force ratio

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Models of two-phase flow: two-fluid model (stratified 2D flow)

- It is useful to write the closure laws in a generic dimensionless form by introducing the friction factor

\[ f_i = -\frac{\tau_i}{\frac{1}{2} \rho_i (U_i - U_g) (U_i - U_l)} \]
\[ f_G = -\frac{\tau_{iG}}{\frac{1}{2} \rho_i (U_i - U_g)} \]
\[ f_L = -\frac{\tau_{iL}}{\frac{1}{2} \rho_i (U_i - U_l)} \]

- If one uses for the Reynolds numbers:

\[ Re_i = \frac{2 \rho_i h_i (U_i - U_L)}{\mu_G} \]
\[ Re_G = \frac{2 \rho_h h_G (U_G - U_L)}{\mu_G} \]
\[ Re_L = \frac{4 \rho_i h_i (U_G - U_L)}{\mu_L} \]

- Then the closure laws take the same form

\[ f_i = -\frac{24}{Re_i}, \quad f_G = -\frac{24}{Re_G}, \quad f_L = -\frac{24}{Re_L} \]

Models of two-phase flow: two-fluid model (stratified flow)

- The closure laws are written in the generic dimensionless form

\[ f_i = -\frac{\tau_i}{\frac{1}{2} \rho_i (U_i - U_g) (U_i - U_l)} \]
\[ f_G = -\frac{\tau_{iG}}{\frac{1}{2} \rho_i (U_i - U_g)} \]
\[ f_L = -\frac{\tau_{iL}}{\frac{1}{2} \rho_i (U_i - U_l)} \]

- The Reynolds numbers are defined as follows:

\[ Re_i = \frac{D_h_i}{\frac{1}{2} h_i v - U_l} \quad \text{where} \quad D_h_i = \frac{4A_i}{S_i + S_l} \]
\[ Re_G = \frac{D_h_i}{\frac{1}{2} h_G v} \]
\[ Re_L = \frac{D_h_i}{\frac{1}{2} h_L v} \quad \text{where} \quad D_h_i = \frac{4A_i}{S_i} \]
Models of two-phase flow:
two-fluid model (recipes for stratified flow)

- Wall shear stresses
  \[ f_w = 3.48 - 4 \log \left( \frac{2 \frac{k}{D}}{\frac{Re_k}{v_R}} \right) \]
  with \( Re_k = \frac{vkD_i}{\nu_k} \)
  \[ D_{hi} = \frac{4 \frac{R_i}{A}}{\frac{S_{hi}}{S_R} + \frac{1}{S_i}} \]

- Interfacial shear stress
  \[ \frac{f_i}{f_{conel}} = 1 - 15 \left( \frac{1}{D} \sqrt{\frac{\frac{1}{k_1}}{\frac{1}{k_2}}} - 1 \right) \]
  \( Hanratty \ et \ al \)

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